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BAYESIAN AND EVT VALUE-AT-RISK ESTIMATES OF INDIA'S NON-FINANCIAL FIRMS

ABSTRACT

The Companies Act 2013 has made it mandatory for firm's Board of Directors Report to include a statement indicating elements of risk faced by companies. In the IMF report of March 2015, it is mentioned that India's non-financial company's external commercial borrowings rose by 107% between March 2010 to March 2014. The stress test based on exchange rate and profits demonstrated continuing high vulnerabilities of the firms. Looking at both the important factors, the current study estimates the Value-at-Risk (VaR) of 106 non-financial Indian firms. It is well a documented fact that return series is non-normal, therefore taking bivariate distribution of return and foreign exchange rate. VaR is calculated using the extreme value theory method and Bayesian method. The results suggest that Bayesian method provides the best VaR estimates

Key Words: non-normality, extreme value theory (EVT), value at risk (VaR), Bayesian VaR, GARCH

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INTRODUCTION

The IMF report of March 2015 documented that India's non-financial companies' external commercial borrowings rose by 107% between March 2010 and March 2014. This leads to increase in foreign exchange risk exposure of the non-financial firms. Although external commercial borrowings is a cheaper source of debt, increased exposure of foreign exchange risk is offsetting this benefit. The Reserve Bank of India's Executive Director G. Mahalingam, in his address as the keynote speaker on February 27 2015, mentioned that the un-hedged corporate exposure remains a major risk factor for the non-financial firms.

Prior to the East Asian crisis of 1997, non-financial companies in many of the East Asian countries accumulated large stocks of unhedged foreign currency borrowings (FCB). These firms experienced credit distress resulting from large unexpected depreciation of the currency. To prevent repeating of situation like East Asian crisis of 1997, there must be a mechanism to check the increasing foreign exchange risk exposure of firms on a continuous basis.

The multivariate VaR can act as a method of continuous quantification of firms' downside risk exposure taking into consideration many risk factors together. Apart from this, if continuous evaluation of exposure is done, then it would be easy for policy makers to decide the minimum hedge ratio for the firms to determine which firm has greater exposure to foreign exchange risk. As per the current scenario RBI directs the firms to hedge foreign exchange exposure due to external commercial borrowings (ECB), but no minimum hedge ratio is implemented or made compulsory.

The Section 134 of the Companies Act 2013 also documents that the Board of Directors' report must include a statement indicating development and implementation of a risk management policy for the company. Hence, looking at both the factors of increasing unhedged foreign currency exposure of Indian non-financial firms and section 134 of the Companies Act 2013 where risk reporting should be a part of annual statements of the companies. This study focuses on the reporting of the downside risk taking into consideration the foreign exchange exposure of firms with the help of various VaR models. Hirtle (2007) documented that 24 US holding banks reported VaR in their financial statements. Hence, VaR can act as the prospective tool to report downside risk in non-financial firms annual statements. The importance of risk factors identification and reporting as well as increased financing of debt through ECB by Indian non-financial companies needs to be highlighted.

Therefore, the current study estimates the downside risk of the select India's non-financial firms with the help of VaR methodology using Extreme value theory and Bayesian VaR methodology in bivariate setting. VaR can act as an imperative tool in market risk factors quantification and market risk factors reporting of the firms. This paper is organized into following sections: Section 2 documents the literature review. Section 3 covers methodology and data used, and Section 4 presents the empirical results, and Section 5 concludes the study.

LITERATURE REVIEW

In the existing literature, foreign currency exposure of the firms can be estimated by using market model. Adler and Dumas (1984) estimated that foreign exchange exposure of firm i as the value of β_i in the following augmented capital asset pricing model (CAPM): $R_{it} = a_i + g_i R_t M + b_i R_t FX + e_{it}$ in which R_{it} is firm i 's stock return, $R_t M$ is the market return, and $R_t FX$ is the percentage change in the trade-weighted nominal exchange rate (an increase indicates an appreciation) The value of β_i can be interpreted as firm i 's foreign exchange exposure of net financial and operational ("natural") hedging, after accounting for market conditions. However, CAPM has an assumption of linear relationship. This linear assumption is not always valid. Secondly, it has an assumption of normality of the disturbance terms and homoscedasticity of error term.

Hsieh (1988) and Meese (1986) argued that the distribution of returns on equities and other assets is typically leptokurtic, that is, the unconditional return distribution shows high peaks and fat tails as well it has heteroscedasticity. It is observed that due to frequent shocks caused by various macroeconomic factors, there is significant skewness and kurtosis in the return series of stocks and stock indices (Bekaert et al. 1998). Therefore, this study searched for better tools like VaR. In this study, VaR is estimated using the conditional EVT in bivariate setting which can estimate the downside risk of the company as well as indirect exposure to external risk factors like foreign exchange risk. Hence, methodology proposed by McNeil and Frey (2000) was selected in this study.

McNeil and Frey (2000) proposed a methodology to estimate the VaR that uses the extreme value theory (EVT) with volatility models, known as the conditional EVT. It is well documented that stock return series exhibit problem of heteroscedasticity and, therefore, in

this paper the author has combined GARCH models with EVT to estimate the distribution of tails.

Bystrom (2004) applied both unconditional and conditional EVT models to the stock market and found that conditional EVT models provide accurate VaR estimates. Bekiros and Georgoutsos (2005) compared the various VaR models, with a special emphasis on EVT, peak over threshold (POT) and block maxima (BM) methodology. Their results confirmed that EVT (POT) method outperformed other traditional VaR models. Tolikas, Koulakiotis, and Brown (2007) compared EVT with traditional measures (e.g., parametric method, historical simulation, and Monte carlo) reported that EVT outperformed rest of the traditional methods, especially at very high confidence levels. According to their results, historical simulation methods performs equally well as EVT. Danielsson and De Vries (2000) reported that unconditional EVT works better than the traditional historical simulation or parametric approaches when a normal distribution for returns is assumed, and a EWMA model is used to estimate the conditional volatility of the return.

Abad (2014) also documented the superiority of conditional EVT methodology. Comparative studies of VaR models, such as Nozari et al. (2010) showed that conditional EVT approaches perform best with respect to forecasting the VaR. Marimoutou, Raggad Trabelsi (2009) used different models and confirmed that the filtering process of return series was important for obtaining better results. Serfling (2002) reviewed on the multivariate quantile techniques. Chan and Gray (2006) used the EGARCH model for volatility estimation and used EVT to model the tails of the return distribution.

Abad, Benito, and Lopez (2014), Bali and Theodossiou (2007) and Polanski and Stoja (2010) used parametric method with asymmetric, leptokurtic distributions and mixed-distribution. Embrechts and Puccetti (2006) used the method of multivariate quantiles. However, according to them, the above methods were not superior to EVT. Sener et al. (2012) stated that the performance of VaR methods does not depend entirely on whether they are parametric, non-parametric, semi-parametric or hybrid, but rather on whether they can model the asymmetry of the underlying data effectively or not. Allen, Singh, and Powell (2013) and Karmakar (2013) emphasized the superiority of EVT method in VaR estimation.

Majority of the above studies focus on comparison of VaR models estimated with tradition methodology and EVT using univariate series. The current study estimates VaR by taking into consideration other risk factors such as foreign exchange risk in a bivariate setting. Very few studies estimated VaR models with the help of Bayesian method. Papers like Yu and Moyeed (2001), Tsionas (2003), and Geraci and Bottai (2007) used Bayesian

methodology for quantile estimation. From the above studies, it is evident that estimation of VaR using EVT offers major improvements over well-known methods and there is scarce literature on VaR estimation by the Bayesian method, which can prove to be a promising candidate in precise VaR estimation. This is because Bayesian method is preferred even if data is scarce, it uses other sources of information through a prior distribution. Secondly, the output of the Bayesian analysis, which is the posterior distribution, provides a more complete inference. Since, the objective of an extreme value analysis is usually an estimate of the probability of future events reaching extreme levels, expression through predictive distributions is better.

In the current study, the downside risk is calculated taking bivariate distribution of stock return and change in foreign exchange rate. VaR is estimated by three methodologies (1) Conditional bivariate EVT, (2) Conditional bivariate Bayesian VaR, and (3) univariate VaR by fitting skewed-student-*t*-distribution, generalized hyperbolic distribution, and generalized hyperbolic skewed distribution to the return series. Bivariate methodologies take into consideration the other risk factor like foreign exchange risk faced by the firms.

METHODOLOGY

It is a well-documented fact that financial asset returns are non-i.i.d and fat-tailed (McNeil and Frey 2000). Fitting a model taking normal assumptions will not be useful. In this paper, EVT is used to model VaR of stock returns of the firms. The EVT models the maxima or minima of a stochastic variable. There are two ways of modelling extremes of a stochastic variable. In the first approach, the time horizon is divided into blocks or periods and it considers the maximum value in each successive period. These selected observations constitute the extreme events, also called block (or per-period) maxima. However, this method is not particularly suited for financial time series because of volatility clustering. Therefore, extreme events tend to follow one another. As the block maxima method considers only the maximum return in each period, many relevant data points are excluded from the analysis. The second method uses data points above a given high threshold. It is better suited for financial series. Therefore, the POT method has become the method of choice in this study. Fisher and Tippet (1928) theorem gave limiting distribution of sample maxima. For a large class of underlying distribution functions the conditional excess distribution function $F_u(y)$ for a large value of u , is well approximated by $F_u(y) \cong$

$G_{\xi,\beta}(y); u \rightarrow \infty$. Where, $G_{\xi,\beta}$ is the so-called generalized Pareto distribution (GPD). GPD tail estimator is as follows;

$$F(x) = 1 - \frac{N_u}{n} \left(1 + \xi \left(\frac{x-u}{\beta}\right)^{-1/\xi}\right) \quad (1)$$

For $x > u$. For a given probability, $q > F(u)$ the VaR estimate is calculated by inverting the tail estimation formula above to get

$$VaR_q = u + \frac{\beta}{\xi} \left(\frac{n}{N_u} (1 - q)\right)^{-\xi} - 1 \quad (2)$$

Where, VaR_q represents VaR at quantile q , u is the threshold, β is scale parameter, ξ is shape parameter, n is observations above a threshold, and N_u is total number of observations. $\xi > 0$ corresponds to heavy-tailed distributions whose tails decay like power functions, such as the Pareto, Student's t , Cauchy, Burr, log-gamma and Frechet distributions. $\xi = 0$ corresponds to distributions like the normal, exponential, gamma, and lognormal. $\xi < 0$ are short tailed distributions. Best GPD estimator of the excess distribution is obtained by trading bias against variance. We chose n high to reduce the chance of bias while keeping N large i.e. u low to control the variance of the parameter estimates.

EVT hypothesizes that as the threshold u tends to the distributional upper endpoint, the limiting distribution of the excesses must fall in the Generalized Pareto family of distributions. So, whatever the original distribution of the measurements provided, we chose an appropriately high threshold, the distribution of values exceeding that threshold should be well approximated by a GPD. Usual parameterization of GPD is in the form of scale parameter β and ξ shape parameter.

Let X_t is a strictly stationary time series of daily observations of the negative log return of stock price. X is represented as follows:

$$X_t = \mu_t + \sigma_t z_t \quad (3)$$

Where, z_t is white noise process, independent and identically distributed (i.i.d). μ_t is mean of the series and σ_t is variance of the series. Let $F_u(x)$ denote marginal distribution

of X_t . In this paper two step procedures is used in estimating VaR as done by McNeil and Frey (2000). The procedure is as follows;

1. Fit ARMA-GARCH type model to the return of asset. Estimate μ_{t+1} and σ_{t+1} . Extract the standardized residuals of the fitted model.
2. Fit EVT model to the standardized residuals as they are independent and identically distributed and find out the 99% quantile for 1 day ahead VaR estimation.

Mean of the return series is modelled by ARMA model whereas volatility is modelled by GARCH family models. Volatility model fitted the current study are as follows; EGARCH, APGARCH, CsGARCH.

GPD fitting in case of bivariate distribution

Like the GPD model for excesses above a threshold for univariate series, the dependence component of the Heffernan and Tawn (2004) model also conditions on a variable exceeding a threshold. It then described the conditional distribution of the remaining variables given the threshold excess by the first variable, using a regression type model. The regression type dependence model is defined not on the original data scale, but after marginal transformation to standardized margins.

Let $x = x_1, x_2, \dots, x_n$ be a d dimensional random variable with arbitrary marginal distributions. Let F_i denote an estimate of the i_{th} marginal distribution function. ($i = 1, \dots, d$) and let G denote the distribution function of the standardized marginal distribution to be determined. The vector variable X is transformed to y_1, y_2, \dots, y_d , a variable having standardized marginal distributions.

$$Y_i = (G^{-1}(\widehat{F}_i(X_i)), i, \dots, d).$$

In practice, the \widehat{F}_i is the marginal empirical distribution functions of the data or the semi-parametric model using the empirical distributions. It fits below a threshold and GPD model is fitted to the tails of the distributions above the threshold.

In case of joint distribution, joint likelihood is used:

$$l(\theta) = \sum_{I_{1,2}} \log g(z_{1,i}, z_{2,i}) + \sum_{I_1} \log g_1(z_{1,i}) + \sum_{I_2} \log g_2(z_{2,i})$$

$I_{1,2}, I_1, I_2$ denote the daily returns. g_1, g_2 are the marginal densities of how g is exploited. In the case of EVT, the parameters are estimated by three methods (1) maximum likelihood estimate (MLE), it is the most widely used method, (2) Methods of moment (MOM), and 3) the Bayesian method. MLE has the shortcoming that it is valid only for certain values of the parameters. Hosking and Wallis (1987) documented that the algorithm used for calculation of MLE fail to converge. According to Castillo and Hadi (1997), MOM is a simpler approach, but it can result in increased sampling errors due to the squaring of observations and hence it is not frequently used in literature.

De Zea and Turkman (2003) used the Bayesian method in estimation and proved that it gives precise parameter estimates as compared to log likelihood. In the POT method, Bayesian is preferred even if the data is scarce. This is because it uses other sources of information through a prior distribution. Secondly, the output of a Bayesian analysis, the posterior distribution, provides a more complete inference than the corresponding maximum likelihood analysis. Since the objective of an extreme value analysis is usually an estimate of the probability of future events reaching extreme levels, expression through predictive distributions is better.

Posterior distribution is estimated by simulation using Monte Carlo Markov chain model (MCMC). According to Coles and Tawn (1996), the Bayesian method is favored as compared to likelihood because it violates regularity condition. This violation of the usual regularity conditions means that the standard asymptotic likelihood results are not automatically applicable. When $\xi > -0.5$ maximum likelihood estimators are regular, in the sense of having the usual asymptotic properties. While when $-1 < \xi < -0.5$ then maximum likelihood estimators are generally obtainable, but do not have the standard asymptotic properties. But when $\xi < -1$ maximum likelihood estimators are unlikely to be obtainable. This problem is not encountered in the Bayesian estimator.

Bayesian estimation

In the Bayesian estimation, we assume data $x = x_1, x_2, \dots, \dots, x_n$ as the realizations of a random variable whose density falls within a parametric family $\mathcal{F} = \{f(x; \theta): \theta \in \Theta\}$.

However, it is assumed that it is possible to formulate beliefs about θ , without reference to the data, that can be expressed as a probability distribution. For example, if we are sure that $0 \leq \theta \leq 1$, that any value in that range is equally likely, this could be expressed by the probability distribution $\theta \sim U(0,1)$. A distribution on the parameter made without reference to the data, is termed a prior distribution. In the Bayesian setting, parameters are treated as random variables, and the prior distribution consists of parameter distribution prior to the inclusion of additional information provided by data likelihood for θ as $f(x|\theta)$.

For example, if the X_i are independent, then:

$$f(x|\theta) = \prod_{i=1}^n f(x_i; \theta)$$

Bayes' Theorem states:

$$f(\theta|x) = \frac{f(\theta)f(x|\theta)}{\int f(\theta)f(x|\theta)d\theta} \quad (4)$$

In this initial set of beliefs as represented by the prior distribution $f(\theta)$ is used to formulate posterior distribution, $f(\theta|x)$, that includes the additional information provided by the data x . We can simulate from the posterior distributions of the parameters. Bayesian estimation is based on MCMC. Metropolis algorithm is used to simulate from the joint posterior distribution of the parameters.

Data

The period of analysis is considered from January 3, 2011 until September 20, 2016. This period is considered in the study as foreign exchange risk exposure has increased since 2011 due to ECB by the Indian non-financial firms. 106 companies daily adjusted closing price data is taken for analysis. Daily adjusted closing prices of stocks are taken from the Bloomberg database. Data is taken for companies listed on the BSE500 Index. Real effective exchange rate (REER) is taken as foreign exchange rate. To calculate the volatility of the REER GARCH (1,1), model with generalized hyperbolic distribution for innovation is used.

It is a well-known fact that return series are not i.i.d. Therefore, the mean of the return series is modelled by ARMA methodology and volatility is modelled by GARCH family models. Best fit AR and MA terms and GARCH model is assessed by information criterion and log-likelihood. Thereafter, standardized residuals ($z_t = \frac{r_t - \mu_t}{\sigma_t}$) of the model is extracted which are i.i.d and best suited for fitting the GPD model.

RESULTS

Descriptive Statistic

It is evident from Table 1 that return series is skewed and it has high kurtosis. If we look at the Jarque-Bera test, it is evident that return series is not normally distributed. Augmented Dicky-Fuller test confirms that the return series is stationary, as null hypothesis of unit root is rejected in all the cases. It is a well-documented fact that returns on asset show heteroscedasticity, it is evident from the ARCH-LM test that all the return series has heteroscedasticity and therefore first returns of stock prices are modelled by GARCH family. Results for the first ten companies are shown for brevity.

Table 1. Summary Statistic of the Return Series of Select Companies

Descriptive Statistic										
Statistic	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
Mean	0.001	0.000	0.000	-0.001	0.001	0.000	0.003	0.001	0.000	0.001
Median	0.000	-0.002	0.000	-0.001	-0.002	-0.001	0.001	0.000	0.000	0.000
Maximum	0.153	0.190	0.063	0.206	0.182	0.132	0.182	0.097	0.128	0.102
Minimum	-0.096	-0.091	-0.066	-0.240	-0.137	-0.105	-0.174	-0.095	-0.119	-0.148
Std. Dev.	0.019	0.021	0.016	0.032	0.030	0.024	0.026	0.016	0.025	0.020
Skewness	1.143	1.587	0.115	-0.131	1.071	0.549	0.795	0.861	0.300	-0.121
Kurtosis	10.589	13.422	3.982	8.459	7.836	5.494	10.024	9.266	5.068	7.630
Jarque-Bera	3578.102	6760.702	57.898	1701.529	1593.464	422.931	2954.482	2405.451	264.162	1224.144
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ADF Test	-34.09***	-33.0***	-35.2***	-35.8***	-36.1***	-36.1***	-34.2***	-37.0***	-34.8***	-24.5***
ARCHLM	2.5***	5.6***	8.6***	9.9***	11.8***	4.2***	3.2***	5.5***	7.5***	58.4***

Note: ADF stands for Augmented Dicky Fuller test, which has null hypothesis that series has unit root. Time Period considered for the analysis is from January 3rd, 2011 till September 20th, 2016. ARCH test is used for checking heteroskedasticity in data, which has null hypothesis of homoscedasticity. ***,** denote significance at 1% and 5% respectively

Table 2 represents ARMA-GARCH model fitted to the return series. Standardized residuals are extracted for EVT fitting.

Table 2. ARMA-GARCH Model Fitted to Return Series

S.No.	GARCH Model	ARMA Terms	Error Distribution	AIC	Log-Likelihood
1	ApARCH(1,1)	ARMA(1,1)	GHYP	-5.5	2670.9
2	ApARCH(1,1)	ARMA(2,1)	GHYP	-5	2465
3	eGARCH(1,1)	ARMA(1,1)	GHYP	-5.4	2632.3
4	eGARCH(1,1)	ARMA(1,1)	GHYP	-4.31	2094
5	ApARCH(1,1)	ARMA(1,2)	GHYP	-4.5	2205.8
6	ApARCH(1,1)	ARMA(1,1)	GHYP	-4.6	2273.6
7	ApARCH(1,1)	ARMA(1,1)	GHYP	-4.61	2243
8	ApARCH(1,1)	ARMA(1,1)	GHYP	-5.8	2816
9	ApARCH(1,1)	ARMA(1,1)	GHYP	-4.58	2229
10	ApARCH(1,1)	ARMA(1,1)	GHYP	-5.2	2526

Note: GHYP= Generalized hyperbolic distribution

In the present study, data is divided into two parts, in first part model parameters are estimated and in-sample VaR is calculated; the second part, model testing (out-of-sample VaR is calculated) is done. From total return series, last 400 observations are left for out of sample testing. Parameter estimated from the estimation window of length n is used to estimate one step ahead VaR values over next 400 observations. From the parameter estimates, the next interval VaR is computed. In other words, keeping the size of the window n constant, the estimation procedure is rolled forward one interval and repeated to calculate the next interval VaR.

The main advantage of this rolling window technique is that it allows us to capture dynamic time-varying characteristics of the data in different time periods. As documented by McNeil and Frey (2000), once mean and volatility using ARMA-GARCH model is calculated both in-sample and forecasted, EVT model as explained in methodology section is used for modelling tails of the distribution. The conditional VaR estimate is then obtained by replacing the POT quantile with $VaR_{t+h}^\alpha = \mu_{t+h} + \sigma_{t+h} Q_\alpha(Z)$. Where, VaR_{t+h}^α is VaR forecasted h period ahead with α level of significance. μ_{t+h} is h period ahead mean, σ_{t+h} is h period ahead volatility and is $Q_\alpha(Z)$ α quantile of returns distribution estimated by fitting POT method to the tails of the distribution. The most important step in fitting EVT model to the return series is to select the appropriate threshold(u) above which Pareto

distribution is fitted. Following methods are used in the present study to select the threshold (n).

Threshold selection

Threshold stability plots, mean residual life (MRL) plots, and quantile-quantile plot are used for threshold selection. The point is chosen as the threshold where MRL plot should be linear. From Figure 2 it is evident that threshold of 1.4 is optimal. The sign of the gradient in the linear part of the MRL plot. It corresponds to the sign of the shape parameter, and hence indicates the shape of the tail (e.g., negative slope shows a short-tailed distribution, a horizontal line (e.g., zero gradient) shows an exponential type tail and a positive slope suggests a heavy tailed distribution).

Quantile-Quantile plot

Ferro and Segers (2003) "propose a quantile-quantile (Q-Q) plot of observed normalized inter-exceedance times against standard exponential quantiles to diagnose model fit. It's appearance is slightly different from a standard Q-Q plot (which shows the data hugging a straight line in the case of a good model fit). In Q-Q plot, the underlying model is a mixture of degeneracy at zero and the exponential distribution, in this broken stick shape is identified. The quantile is indicated with a vertical line and above this line we look for the observed and theoretical quantiles hugging a straight line. Below this line we look for a sudden attenuation of the observed times close to zero." From Figure 1, it is evident that threshold of 1.4 is the best fit for the return series.

Figure 1. Q-Q plot of Company 1

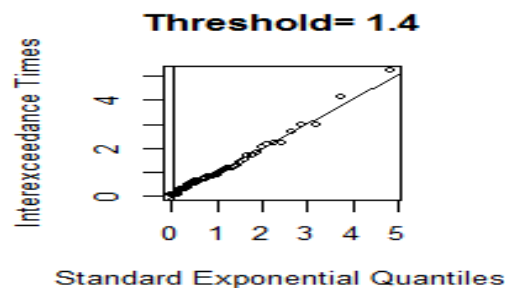
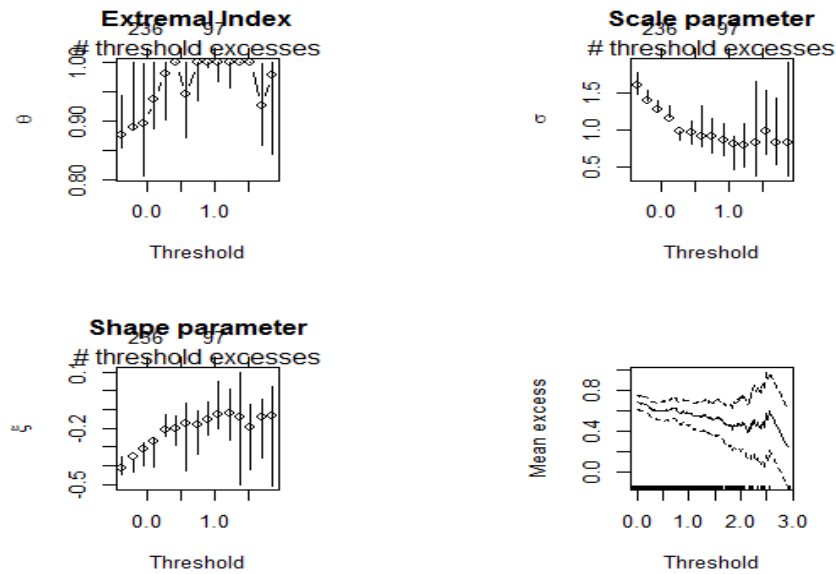


Figure 2. Threshold Stability Plot and Mean Excess Function of Company 1



In TSP generalized Pareto distribution function is fitted to the data with sequence of threshold choices along with some variability information. Parameters are estimated using a range of thresholds. That threshold is chosen where parameter estimates in threshold stability plots is constant above the chosen threshold. That means once a threshold is high enough, raising the threshold further should not dramatically change the estimated value. From Figure 2 it is evident that threshold of 1.4 is good for the Company1.

Since, we have bivariate distribution hence GPD is fitted to bivariate distribution using methodology mentioned in methodology section. The other method used for tails estimation is by Bayesian methodology as explained in methodology section. GPD model fit to the given threshold is deduced from Figure 3. Whereas, Bayesian model fit is deduced from the Figure 4. It is evident from the figure that the plots of the Markov chains ought to look fat hairy caterpillars, that means algorithm has converged on its target distribution and is a good fit of model.

Figure 3. Model Fit Diagnosis

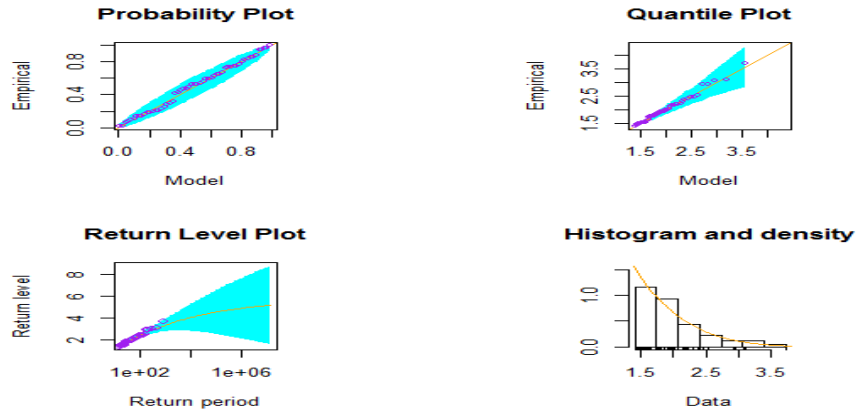


Figure 4. Model Fit Diagnosis for Company 1's Bayesian VaR Estimate

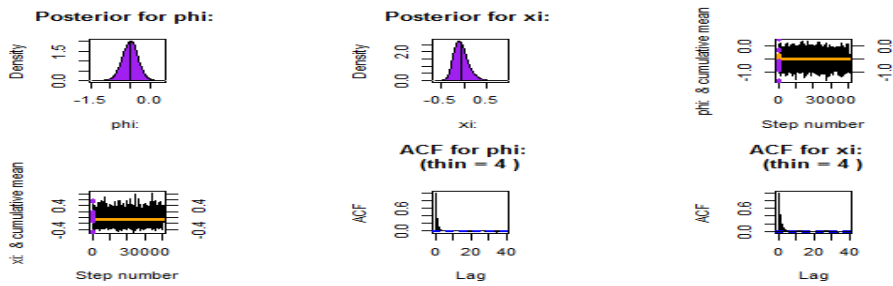


Table 3. EVT and Bayesian VaR Parameter Estimates

S.no	Threshold	Rate of exceedances	Extreme Value Theory				Bayesian VaR			
			Model Log likelihood	AIC	Phi(Scale)	Clusters	ξ	Phi(Scale)	ξ	
1	1.4	0.06	-22.00	48.60	0.451** [0.186]	55	-0.143 [0.13]	0.50	-0.06	
2	1.2	0.07	-19.00	42.90	0.70*** [0.15]	65	0.005 [0.08]	0.72	0.06	
3	1.5	0.05	-16.00	36.10	0.5** [0.19]	52	-0.19 [0.13]	0.54	-0.12	
4	1.1	0.09	-51.00	107.00	0.662** [0.15]	99	0.247 [0.11]	0.68	0.29	
5	1.1	0.08	-22.00	48.50	0.65** [0.15]	99	-0.043 [0.10]	0.68	-0.05	
6	1	0.09	-43.00	90.30	0.663** [0.17]	85	0.171 [0.14]	0.69	0.23	
7	1.3	0.04	-19.00	42.60	0.61** [0.20]	42	0.07 [0.14]	0.65	0.17	
8	1.1	0.07	-20.00	44.20	0.92*** [0.19]	68	0.224 [0.22]	0.97	0.30	
9	1.5	0.05	-15.00	34.70	0.76** [0.23]	47	0.09 [0.18]	0.82	0.20	
10	1.4	0.06	-17.00	38.30	1.059*** [0.22]	42	0.34* [0.18]	1.10	0.43	

Table 3 gives the VaR parameter estimates. Where, Φ represents scale parameter and ξ represents shape parameter. It is observed that material, capital goods, and automobiles and components are high-risk sectors, whereas pharmaceuticals and biotechnology and life sciences are low risk sectors. Table 4 shows the average VaR values calculated. The correlation of VaR values and total debt of non-financial companies is also calculated. Correlation comes out to be positive at 0.51. That means firms that have high debt face higher downside risk. This question the hedging done by firms to mitigate the increased foreign exchange exposure due to foreign debt in the balance sheet.

Table 4. Average VaR Values Calculated Using the EVT Method

S.#	Companies	VaR	S.#	Companies	VaR	S.#	Companies	VaR	S.#	Companies	VaR
1	3M INDIA LTD	0.05	28	BAJAJ HINDUSTHAN	0.05	55	ELGI EQUIPMENTS	0.06	82	EMAMI LTD	0.047
2	ABB INDIA LTD	0.067	29	BALMER LAWRIE	0.04	56	ENGINEERS INDIA	0.05	83	HMT LTD	0.102
3	ACC LTD	0.057	30	BLUE STAR LTD	0.03	57	ESCORTS LTD	0.07	84	HINDALCO INDS	0.08
4	ADANI ENTERPRISE	0.071	31	BHARAT PETROL	0.06	58	EXIDE INDUS LTD	0.06	85	HINDUSTAN PETRO	0.062
5	AEGIS LOGISTICS	0.084	32	BALRAMPUR CHINI	0.05	59	FAG BEARINGS	0.04	86	HATSUN AGRO PROD	0.055
6	ASAHI INDIA GLAS	0.059	33	BERGER PAINTS	0.08	60	FORCE MOTORS LTD	0.06	87	HINDUSTAN UNILEV	0.042
7	AJANTA PHARMA	0.088	34	BRITANNIA INDS	0.05	61	FINOLEX CABLES	0.09	88	HONEYWELL AUTOMA	0.062
8	AKZO NOBEL INDIA	0.048	35	BAYER CROPSCIENC	0.06	62	FINOLEX INDUS	0.06	90	INDRAPRASTHA GAS	0.051
9	ASHOK LEYLAND	0.09	36	CCL PRODUCTS IND	0.05	63	GAIL INDIA LTD	0.07	91	INDIAN HOTELS CO	0.044
10	APOLLO HOSPITALS	0.064	37	CONTAINER CORP	0.04	64	GODREJ CONSUMER	0.05	92	INFOSYS LTD	0.035
11	ASIAN PAINTS LTD	0.032	38	CADILA HEALTHCAR	0.04	65	GODREJ INDUSTRIE	0.06	93	INGERSOLL RAND	0.057
12	APOLLO TYRES LTD	0.081	39	CENTURY TEXTILE	0.08	66	GEOMETRIC LTD	0.06	94	INDIAN OIL CORP	0.044
13	AUROBINDO PHARMA	0.073	40	CESC LTD	0.06	67	GILLETTE INDIA	0.04	95	IPCA LABS LTD	0.048
14	AARTI INDUS LTD	0.091	41	CHAMBAL FERTILIS	0.07	68	GLAXOSMITHKLINE	0.04	96	ITC LTD	0.027
15	ARVIND LTD	0.069	42	CIPLA LTD	0.04	69	GUJARAT NARM VLY	0.05	97	JB CHEMS & PHARM	0.057
16	ATUL LTD	0.066	43	CROMPTON GREAVES	0.07	70	GLENMARK PHARMA	0.03	98	JINDAL STEEL & P	0.066
17	BASF INDIA LTD	0.046	44	COROMANDEL INTER	0.06	71	GODFREY PHILLIPS	0.06	99	JUBILANT LIFE	0.051
18	BOMBAY BURMAH TR	0.091	45	CYIENT LTD	0.05	72	GRASIM INDS LTD	0.03	100	KAJARIA CERAMICS	0.048
19	BEML LTD	0.062	46	DABUR	0.04	73	GREAVES COTTON	0.07	101	CUMMINS INDIA	0.027
20	BF UTILITIES LTD	0.125	47	DCM SHRIRAM LTD	0.07	74	GUJARAT STATE F	0.04	102	KANSAI NEROLAC P	0.049
21	BHARTI AIRTEL	0.049	48	DEEPAK FERTIL	0.05	75	GATI LTD	0.09	103	KPIT TECHNOLOGIE	0.06
22	BHARAT ELECTRON	0.051	49	DISH	0.09	76	HAVELLS INDIA	0.05	104	KALPATARU POWER	0.066
23	BHARAT HEAVY ELE	0.055	50	DIVI LABS LTD	0.04	77	HINDUSTAN CONST	0.04	105	KRBL LTD	0.068
24	BHARAT FORGE CO	0.063	51	DR REDDY'S LABS	0.03	78	HCL INFOSYSTEMS	0.06	106	LAKSHMI MACHINE	0.042
25	BALKRISHNA INDS	0.064	52	EID PARRY INDIA	0.07	79	HCL TECH LTD	0.04			
26	BIOCON LTD	0.047	53	EIH LTD	0.05	80	HEXAWARE TECHNOL	0.06			
27	BAJAJ ELECTRICAL	0.048	54	EICHER MOTORS	0.06	81	HERO MOTOCORP	0.04			

Correlation between VAR values and Total debt
=0.518*

Testing VaR fit

In the present study, VaR is estimated by fitting GPD and Bayesian method. Then, VaR accuracy is checked by back-testing the model in the last 400 observations left in the estimation window. For back-testing, the following methodology is used: We have to compare the *ex ante* VaR forecasts with the ex post realized returns. Consider a sequence of past VaR forecasts and a sequence of realized returns. Here, Christoffersen's back testing procedure is used. We started by defining the hit sequence I_t of VaR violations where;

$$I_t = \begin{cases} 1 & \text{if } r_t < -VaR_t \\ 0 & \text{if } r_t \geq -VaR_t \end{cases}$$

Detailed explanation of the back-testing is provided in Appendix 1

The results of back-testing is reported in Table 5 where only results of the first ten companies are reported for brevity from the results of back-testing it is evident that out of 106 companies. The EVT model for bivariate distribution is accepted 57 times in case of in-sample and 45 times in case of out of sample testing of the model. For Bayesian methodology, VaR model is accepted 77 times in case of in-sample and 75 times in case of out of sample testing. While in case of last method, VaR model is accepted 51 times in case of in-sample. Hence, it is evident that Bayesian method is performing best in case of VaR estimation both in sample and out of the sample.

Table 5. Backtesting Results of the Conditional VaR

Conditional VaR							
	Expected Exceedances	Actual Exceedances	LRUC	P-Value	Decision	LRCC critical	LRCC(Ind) P-Value
1	9	1	12.88	0.00	Reject H0	5.99	0.0
2	9	1	12.88	0.00	Reject H0	5.9	0.0712
3	9	1	12.88	0.00	Reject H0	5.99	0.0812
4	9	16	3.4	0.06	Fail to Reject H0	5.99	0.0812
5	9	2	9.09	0.0	Reject H0	5.99	0.0912
6	9	11	0.17	0.67	Fail to Reject H0	5.99	0.0012
7	9	1	12.88	0.0	Reject H0	5.99	0.0012
8	9	1	12.88	0.0	Reject H0	5.99	0.0912
9	9	1	12.88	0.0	Reject H0	5.99	0.0012
10	9	4	4.31	0.03	Reject H0	5.99	0.0012
Bayesian VaR Estimates							
	Expected Exceedances	Actual Exceedances	LRUC	P-Value	Decision	UC-critical	UC(Ind) P-Value
1	9	1	12.88	0.0003	Reject H0	5.99	0.0012
2	9	2	9.09	0.0025	Reject H0	5.99	0.0712
3	9	1	12.88	0.0003	Reject H0	5.99	0.0812
4	9	16	3.49	0.061	Fail to Reject H0	5.99	0.0812
5	9	2	9.09	0.002	Reject H0	5.99	0.0912
6	9	12	0.52	0.467	Fail to Reject H0	5.99	0.0912
7	9	1	12.88	0.0003	Reject H0	5.99	0.0012
8	9	1	12.88	0.0003	Reject H0	5.99	0.0912
9	9	5	2.766	0.096	Fail to Reject H0	5.99	0.0912
10	9	5	2.76	0.096	Fail to Reject H0	5.99	0.0912
Generalized hyperbolic distribution for Innovation							
	Expected Exceedances	Actual Exceedances	LRUC	LRP	Decision	UC critical	UC(Ind) P-Value
1	9	2	9.09	0.002	Reject H0	5.99	0.0012
2	9	2	9.09	0.002	Reject H0	5.99	0.0712
3	9	7	0.823	0.364	Fail to Reject H0	5.99	0.0812
4	9	8	0.309	0.577	Fail to Reject H0	5.99	0.0812
5	9	1	12.88	0.0003	Reject H0	5.99	0.0912
6	9	3	6.363	0.011	Reject H0	5.99	0.0012
7	9	1	12.88	0.0003	Reject H0	5.99	0.0012
8	9	1	12.88	0.0003	Reject H0	5.99	0.0912
9	9	5	2.766	0.096	Fail to Reject H0	5.99	0.0012
10	9	7	0.8237	0.364	Fail to Reject H0	5.99	0.0012

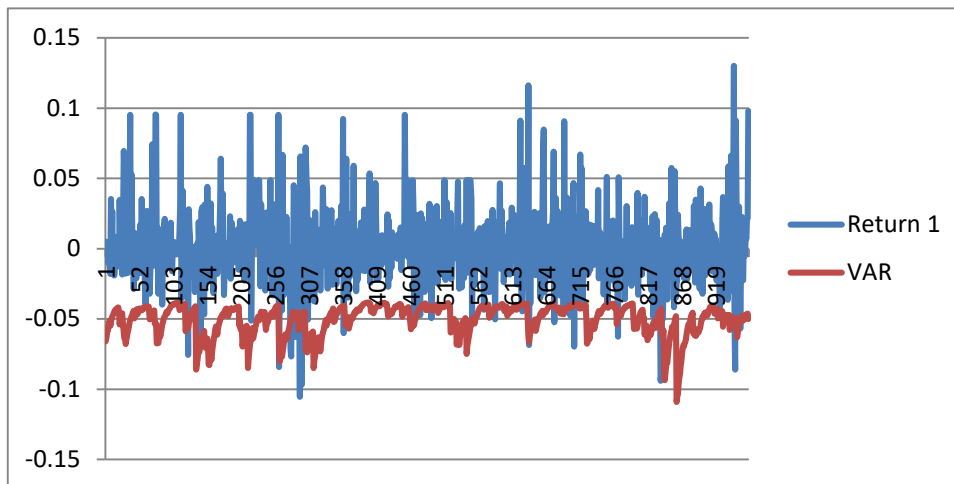
Figure 5. Rolling VaR estimates vis-à-vis returns over 2011 to 2014

Figure 5 reported the rolling 1 day ahead VaR and returns of Company 1 from 2011 to 2014. It is evident from the above figure that VaR values are very closely fitting return series.

CONCLUSIONS

The Companies Act of 2013 lays stress upon the importance and need of managing risk and compliance for the organizations so that they can be in better position to handle and mitigate economic uncertainties and be proactively aware of the highs and lows of business to reap opportunity. The IMF report of March 2015 documented that India's non-financial companies external commercial borrowings rose by 107% from March 2010 to March 2014.

Hence, looking at importance of increasing unhedged foreign currency exposure of Indian non-financial firms and section 134 of Companies Act of 2013, the current study focuses on the reporting of the downside risk of the India's non-financial firms in multivariate VaR form by taking foreign exchange exposure into consideration. Hence, VaR can act as prospective tool to report downside risk in non-financial firm's annual statements. It can act as an indirect way to find out the hedging effectiveness of the non-financial firms.

Adler and Dumas (1984) estimated foreign exchange exposure of firm i as the value of β_i in the following augmented capital asset pricing model (CAPM). This method of estimating foreign exchange exposure suffers from linearity assumption, and normality

assumption of the stock return series. VaR estimation by EVT and Bayesian method in the multivariate form helps in overcoming linearity and normality assumption. Hence, it can act as a precise measure of downside risk estimation.

In the present study, VaR is estimated for 106 non-financial companies using bivariate distribution of foreign exchange rate and stock returns. Three methodologies are used for VaR estimation: (1) extreme value theory (EVT) methodology, (2) Bayesian methodology, and (3) by fitting non-normal distribution to the return series. The results highlight that the Bayesian VaR estimation demonstrates the best downside risk estimate. Accurate VaR estimate can help in precise margin determination in trading, accurate reporting of market risk faced by the firms.

Correlation of VaR values with the total debt is also investigated. Correlation is coming out to be positive at 0.51. That means firms having high debt are facing higher downside risk. This questions the hedging done by firms to mitigate the increased foreign exchange exposure due to foreign debt in the balance sheet.

REFERENCES

- Abad, P., S. Benito, and C. López. 2014. A comprehensive review of value at risk methodologies. *The Spanish Review of Financial Economics* 12 (1): 15-32.
- Adler, M. and B. Dumas. 1984. Exposure to currency risk: Definition and measurement. *Financial management* 41-50.
- Allen, D. E., A. K. Singh, and R. J. Powell. 2013. EVT and tail-risk modelling: Evidence from market indices and volatility series. *The North American Journal of Economics and Finance* 2: 355-369.
- Bali, T. G. and P. Theodossiou. 2007. A conditional-SGT-VaR approach with alternative GARCH models. *Annals of Operations Research* 151 (1): 241-267.
- Bekaert, G., C. B. Erb, C. R. Harvey, and T. E. Viskanta. 1998. Distributional characteristics of emerging market returns and asset allocation. *Journal of Portfolio Management* 24 (2): 102-116.
- Bekiros, S. D. and D. A. Georgoutsos. 2005. Estimation of Value-at-Risk by extreme value and conventional methods: A comparative evaluation of their predictive performance. *Journal of International Financial Markets, Institutions and Money* 15 (3): 209-222.
- Bystrom, H. N. 2004. Managing extreme risks in tranquil and volatile markets using conditional extreme value theory. *International Review of Financial Analysis* 13 (2): 133-152.

- Castillo, E. and A. S. Hadi. 1997. Fitting the generalized Pareto distribution to data. *Journal of the American Statistical Association* 92 (440): 1609-1620.
- Chan, K. F. and P. Gray. 2006. Using extreme value theory to measure value-at-risk for daily electricity spot prices. *International Journal of forecasting* 22 (2): 283-300.
- Christoffersen, P. F. 1998. Evaluating interval forecasts. *International economic review* 39: 841-862.
- Christoffersen, P. and D. Pelletier. 2004. Backtesting value-at-risk: A duration-based approach. *Journal of Financial Econometrics* 2 (1): 84-108.
- Coles, S. G. and J. A. Tawn. 1996. A Bayesian analysis of extreme rainfall data. *Applied Statistics* 45: 463-478.
- Danielsson, J. and C. G. De Vries. 2000. Value-at-risk and extreme returns. *Annales d'Economie et de Statistique* 239-270.
- De Zea Bermudez, P. and M. A. Turkman. 2003. Bayesian approach to parameter estimation of the generalized Pareto distribution. *Test* 12 (1): 259-277.
- Embrechts, P. and G. Puccetti, G. 2006. Bounds for functions of dependent risks. *Finance and Stochastics* 10 (3): 341-352.
- Fisher, R. A. and L. H. C. Tippett. 1928. Limiting forms of the frequency distribution of the largest or smallest member of a sample. *In Mathematical Proceedings of the Cambridge Philosophical Society* 24 (2):180-190.
- Geraci, M. and M. Bottai. 2007. Quantile regression for longitudinal data using the asymmetric Laplace distribution. *Biostatistics* 8 (1): 140-154.
- Heffernan, J. E. and J. A. Tawn. 2004. A conditional approach for multivariate extreme values. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 66 (3): 497-546.
- Hirtle, B. 2007. *Public Disclosure, Risk, and Performance at Bank Holding Companies*. Staff Report: Federal Reserve Bank of New York. No. 293. Federal Reserve Bank of New York: New York, NY.
- Hosking, J. R. and J. R. Wallis. 1987. Parameter and quantile estimation for the generalized Pareto distribution. *Technometrics* 29 (3): 339-349.
- Hsieh, D. A. 1988. The statistical properties of daily foreign exchange rates: 1974–1983. *Journal of International Economics* 24 (1-2): 129-145.
- Karmakar, M. 2013. Estimation of tail-related risk measures in the Indian stock market: An extreme value approach. *Review of Financial Economics* 22 (3): 79-85.
- Marimoutou, V., B. Raggad, and A. Trabelsi. 2009. Extreme value theory and value at risk: application to oil market. *Energy Economics* 31 (4): 519-530.

- Meese, R. A. 1986. Testing for bubbles in exchange markets: A case of sparkling rates?. *Journal of Political Economy* 94 (2): 345-373.
- McNeil, A. J. and R. Frey. 2000. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance* 7 (3): 271-300.
- Nozari, M., S. M. Raei, P. Jahangiri, and M. Bahramgiri. 2010. A comparison of heavy-tailed VaR estimates and filtered historical simulation: Evidence from emerging markets. *International Review of Business Research Papers* 6 (4): 347-359.
- Polanski, A. and E. Stojca. 2010. Incorporating higher moments into value-at-risk forecasting. *Journal of Forecasting* 29 (6): 523-535.
- Sener, E., S. Baronyan, and L. Ali Menguturk. 2012. Ranking the predictive performances of value-at-risk estimation methods. *International Journal of Forecasting* 28 (4): 849-873.
- Serfling, R. 2002. Generalized quantile processes based on multivariate depth functions, with applications in nonparametric multivariate analysis. *Journal of Multivariate Analysis* 83 (1): 232-247.
- Tolikas, K., A. Koulakiotis, and R. A. Brown. 2007. Extreme risk and value-at-risk in the German stock market. *European Journal of Finance* 13 (4): 373-395.
- Tsionas, E. G. 2003. Bayesian quantile inference. *Journal of Statistical Computation and Simulation* 73 (9): 659-674.
- Yu, K. and R. A. Moyeed. 2001. Bayesian quantile regression. *Statistics & Probability Letters* 54 (4): 437-447.

APPENDIX 1

A.1.Backtesting

Testing the unconditional coverage

Zeros are represented by T_0 and number of ones by T_1 . Likelihood ratio test is as follows

$$LR_{uc} = -2\ln\left[\frac{L(1-\alpha)}{L(\hat{\pi})}\right] \quad (A.1)$$

Where, $L(\cdot)$ represents likelihood function of an i.i.d Bernoulli sequence. π is the observed ratio of violations. Replacing with the appropriate function we obtain the expression for the likelihood ratio test.

$$LR_{UC} = -2 \ln \left[\frac{\alpha^{T_0}(1-\alpha)^{T_1}}{(1-T_1/T)^{T_0}(T_1/T)^{T_1}} \right] \sim \chi_1^2 \quad (A.2)$$

which asymptotically has a chi-square distribution.

Testing the independence of the violations

The Christoffersen (1998) test of independence for VaR violations assumes, under the hypothesis of dependence, that the hit sequence can be described by a first-order Markov process with transition probability matrix.

$$\pi_1 = \begin{bmatrix} 1 - \pi_{01} & \pi_{01} \\ 1 - \pi_{11} & \pi_{11} \end{bmatrix}$$

Where, π_{11} is the probability that tomorrow's return is a violation given that today is a violation, and π_{01} is the probability that tomorrow's return is a violation given that today is not a violation. Given a sample of size T the likelihood function of the first-order Markov process is

$$L(\pi_1) = (1 - \pi_{01})^{\pi_{00}} \pi_{01}^{\pi_{01}} (1 - \pi_{11})^{\pi_{10}} \pi_{11}^{\pi_{11}} \quad (A.3)$$

Where $T_{i,j}$ $i,j=0,1$ is the number of observations in the hit sequence with a j following an i . The maximum likelihood estimates of the transition probabilities are;

$$\hat{\pi}_{01} = \frac{T_{01}}{T_{00}+T_{01}} \text{ and } \hat{\pi}_{11} = \frac{T_{11}}{T_{10}+T_{11}}$$

If $T_{11} = 0$ then likelihood function takes following form

$$L(\hat{\pi}_1) = (1 - \hat{\pi}_{01})^{T_{00}} \hat{\pi}_{01}^{T_{01}} \quad (\text{A.4})$$

In the null hypothesis of independence $\pi_{01} = \pi_{11} = \pi$ then transition matrix is

$$\hat{\pi} = \begin{bmatrix} 1 - \hat{\pi} & \hat{\pi} \\ 1 - \hat{\pi} & \hat{\pi} \end{bmatrix}$$

Where, $\hat{\pi}$ is $\frac{T_1}{T}$ is the estimator for the ratio of violations as in the unconditional coverage test. The likelihood function in the case of independence is then given by

$$L(\hat{\pi}_1) = (1 - \hat{\pi})^{\pi_{00} + \pi_{10}} \hat{\pi}^{\pi_{01} + \pi_{11}} \quad (\text{A.5})$$

The likelihood ratio can be used to test the independence hypothesis

$$LR_{Ind} = -2 \ln \left[\frac{L(\hat{\pi})}{L(\hat{\pi}_1)} \right] \sim \chi_1^2$$

Testing the conditional coverage

Christoffersen (1998) tests simultaneously if the number of violations is correct and if the VaR violations are independent. This means testing $\pi_{01} = \pi_{11} = 1 - \alpha$. Christoffersen uses likelihood ratio test

$$LR_{cc} = -2 \ln \left[\frac{L(1-\alpha)}{L(\hat{\pi}_1)} \right] \sim \chi^2_2 \quad (\text{A.6})$$

which has an asymptotic chi-square distribution with two degrees of freedom.

$$LR_{CC} = LR_{uc} + LR_{ind} \quad (\text{A.7})$$